Number and Algebra

IB SL Study Guide

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IB Math AI SL — Number and Algebra

Complete Study Guide

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Topic 1 of the IB Math AI SL syllabus — Paper 1 and Paper 2

IB TIP

Calculator-active course: Unlike Math AA, all AI papers allow GDC use. You still need to show working for “show that” questions, but for most problems your GDC is your best friend. Know how to use the finance solver (TVM), table of values, and equation solver efficiently.

MEMORISE THIS

Core formulas at a glance

Formula	Expression
Percentage error	$\left\lvert \frac{v_A - v_E}{v_E} \right\rvert \times 100\%$
Arithmetic nth term	$u_n = u_1 + (n-1)d$
Arithmetic sum	$S_n = \frac{n}{2}(2u_1 + (n-1)d)$
Geometric nth term	$u_n = u_1 \cdot r^{n-1}$
Geometric sum	$S_n = \frac{u_1(1-r^n)}{1-r},\ r \neq 1$
Compound interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$

Section 1: Scientific Notation, Rounding, and Estimation

1.1 Scientific Notation

Any number can be written in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer.

Real-world context: Scientists measuring cell sizes (3.2×10^{-6} m) or national GDPs (1.8×10^{12} USD) use scientific notation to keep numbers manageable. On Paper 2, data is frequently given in this form.

✎ **WORKED EXAMPLE**

Scientific notation in context

The population of bacteria in a culture doubles every 3 hours. Starting from 500 bacteria, express the population after 24 hours in scientific notation.

After 24 hours there are $\frac{24}{3} = 8$ doubling periods.

$P = 500 \times 2^8 = 500 \times 256 = 128\,000 = 1.28 \times 10^5$

1.2 Significant Figures and Rounding

The IB expects you to give final answers to **3 significant figures** unless told otherwise. Intermediate calculations should carry at least one extra significant figure to avoid rounding error.

Percentage error measures how far an approximation v_A is from an exact value v_E:

$\text{Percentage error} = \left|\frac{v_A - v_E}{v_E}\right| \times 100\%$

⚠ **EXAM ALERT**

Common mistake: Students sometimes use the approximation in the denominator. The IB formula uses the **exact** value in the denominator. This is given in the formula booklet.

✎ **WORKED EXAMPLE**

Percentage error

A student estimates the height of a building as 14.5 m. The actual height is 15.2 m. Find the percentage error.

$\text{Percentage error} = \left|\frac{14.5 - 15.2}{15.2}\right| \times 100\% = \frac{0.7}{15.2} \times 100\% = 4.61\%$

Section 2: Arithmetic Sequences and Series

An **arithmetic sequence** has a constant difference d between consecutive terms. This models linear growth: salary increases by a fixed amount each year, a taxi charges a fixed rate per km.

2.1 The nth Term

$u_n = u_1 + (n - 1)d$

where u_1 is the first term and d is the common difference.

2.2 The Sum of n Terms

$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$

WORKED EXAMPLE

Arithmetic sequence in a real-world scenario

A gym membership costs $40 in the first month, and increases by 5$ *each month. Find the cost in month 12 and the total cost over the first 12 months.*

Here $u_1 = 40, d = 5$.

Month 12: $u_{12} = 40 + 11 \times 5 = 40 + 55 = 95$ dollars.

Total cost: $S_{12} = \dfrac{12}{2}(40 + 95) = 6 \times 135 = 810$ dollars.

2.3 Applications

Arithmetic sequences model situations where a quantity changes by the **same amount** each period:

- Saving a fixed amount each month
- Depreciation by a fixed dollar value
- Seating in a stadium (each row has d more seats than the previous row)

IB TIP

GDC shortcut: On TI-84, enter the sequence formula in Y_1 and use TABLE to see all terms at once. On Casio, use the sequence mode (RECUR). This is much faster than calculating each term by hand.

Section 3: Geometric Sequences and Series

A **geometric sequence** has a constant ratio r between consecutive terms. This models exponential growth and decay: population growth, radioactive decay, compound interest.

3.1 The nth Term

$u_n = u_1 \cdot r^{n-1}$

3.2 The Sum of n Terms

$S_n = \frac{u_1(1-r^n)}{1-r}, \quad r \neq 1$

3.3 Infinite Geometric Series

When $|r| < 1$, the sum to infinity converges:

$S_\infty = \frac{u_1}{1-r}$

WORKED EXAMPLE

Geometric sequence — population decline

A town has 12,000 residents. Each year the population decreases by 3%. Find the population after 10 years and the total number of "person-years" lived over those 10 years.

Here $u_1 = 12000, r = 0.97$.

After 10 years: $u_{11} = 12000 \times 0.97^{10} = 12000 \times 0.7374\ldots = 8849$ (3 s.f.)

Total person-years: $S_{10} = \dfrac{12000(1 - 0.97^{10})}{1 - 0.97} = \dfrac{12000 \times 0.2626}{0.03} = 105\,000$ (3 s.f.)

EXAM ALERT

Be careful with n: If the population **starts** at 12,000, this is u_1. The population after 1 year is $u_2 = u_1 \cdot r$. After 10 years is u_{11}. Many students use $n = 10$ when they should use $n = 11$.

Section 4: Financial Mathematics

Financial maths is a major focus of Math AI. You must understand compound interest and be proficient with your GDC's finance solver (TVM solver).

4.1 Compound Interest

$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$

where:

- FV = future value
- PV = present value (initial investment)
- r = annual interest rate (as a percentage)
- k = number of compounding periods per year
- n = number of years

Compounding	k
Annually	1
Quarterly	4
Monthly	12
Daily	365

WORKED EXAMPLE

Compound interest — comparing options

Anna invests $5000 at 4.2$5000 at 4.3% per annum compounded annually. Who has more after 5 years?*

Anna: $FV = 5000 \times \left(1 + \frac{4.2}{1200}\right)^{60} = 5000 \times 1.0035^{60} = 5000 \times 1.2330 = 6165$ (nearest dollar)

Ben: $FV = 5000 \times \left(1 + \frac{4.3}{100}\right)^{5} = 5000 \times 1.043^{5} = 5000 \times 1.2349 = 6175$ (nearest dollar)

Ben has slightly more. The higher nominal rate outweighs monthly compounding.

4.2 Using the GDC Finance Solver (TVM)

Your GDC has a TVM (Time Value of Money) solver. The key variables are:

Variable	Meaning
N	Total number of payment periods
$I\%$	Annual interest rate
PV	Present value (negative if you pay it)
PMT	Payment per period
FV	Future value
P/Y	Payments per year
C/Y	Compounding periods per year

IB TIP

Sign convention: Money you pay out is **negative**, money you receive is **positive**. For a loan, PV is positive (you receive the money) and PMT is negative (you pay it back).

4.3 Annuities and Amortization

An **annuity** is a series of equal payments made at regular intervals. An **amortization schedule** shows how each payment is split between interest and principal.

WORKED EXAMPLE

Loan repayment

You borrow 20,000 USD at 6% annual interest compounded monthly. You repay in equal monthly instalments over 5 years. Find the monthly payment.

Using the TVM solver: $N = 60, I\% = 6, PV = 20000, FV = 0, P/Y = 12, C/Y = 12$.

Solve for PMT: $PMT = -386.66$ (the negative sign confirms you are paying out).

Monthly payment is **386.66 USD**.

Total paid: $386.66 \times 60 = 23\,200$. Total interest: $23\,200 - 20\,000 = 3200$.

Section 5: Logarithms in Context

Logarithms in Math AI focus on **solving real-world exponential equations** rather than algebraic manipulation.

5.1 The Logarithm as an Inverse

If $a^x = b$ then $x = \log_a b$.

The most common bases are 10 (log) and e (ln).

5.2 Solving Exponential Equations

To find when a quantity reaches a target value, take logarithms of both sides:

$u_1 \cdot r^n = T \implies n = \frac{\ln T - \ln u_1}{\ln r}$

WORKED EXAMPLE

Doubling time

A population of 8000 grows at 2.5% per year. How long until it reaches 16,000?

$8000 \times 1.025^n = 16000$ $1.025^n = 2$ $n = \frac{\ln 2}{\ln 1.025} = \frac{0.6931}{0.02469} = 28.1 \text{ years}$

5.3 Half-Life

The time for a quantity to halve. If r is the decay factor per unit time:

$t_{1/2} = \frac{\ln 0.5}{\ln r}$

WORKED EXAMPLE

Radioactive decay

A sample of 500 g of a radioactive substance decays by 8% per year. Find the half-life and the mass after 15 years.

Decay factor $r = 0.92$.

Half-life: $t_{1/2} = \dfrac{\ln 0.5}{\ln 0.92} = \dfrac{-0.6931}{-0.08338} = 8.31$ years.

After 15 years: $500 \times 0.92^{15} = 500 \times 0.2863 = 143$ g (3 s.f.)

Section 6: Practice Questions

Paper 1 Style (Short Answer)

▶ **Q1.** The 5th term of an arithmetic sequence is 22 and the 12th term is 57. Find u_1 and d.

▶ **Q2.** Express 0.000427 in scientific notation and find the percentage error when it is rounded to 4.3×10^{-4}.

▶ **Q3.** A car purchased for 25,000 depreciates by 15% each year. Find its value after 6 years.

Paper 2 Style (Extended Response)

▶ **Q4.** Sophia invests 8000 at 3.8% per annum compounded quarterly. (a) Find the value after 10 years. (b) Find when the investment first exceeds 12,000. (c) If instead she adds 100 per quarter, use your GDC to find the value after 10 years.

▶ **Q5.** A bouncing ball is dropped from 2 metres. After each bounce it reaches 72% of its previous height. (a) Find the height after the 5th bounce. (b) Find the total vertical distance the ball travels before it stops.

⚠ EXAM ALERT

Exam strategy for financial maths: Always write down the TVM values you enter, even if you solve on the GDC. The IB expects to see $N = ...$, $I\% = ...$, $PV = ...$, $PMT = ...$, $FV = ...$, $P/Y = ...$, $C/Y = ...$ before the answer. Without this, you lose method marks.

May 2026 Prediction Questions

⚠ EXAM ALERT

These are NOT official IB questions. These are trend-based practice questions written to reflect the topic areas and question styles most likely to appear on the May 2026 IB Math AI SL Paper 2. Based on recent exam patterns (2022–2025), expect heavy weighting on: compound interest, annuities (TVM solver), geometric series in real-world contexts, and logarithms in exponential growth/decay scenarios.

WORKED EXAMPLE

Question 1 — Financial Mathematics (TVM Solver) [~8 marks]

Priya takes out a car loan of $18,500 at an annual interest rate of 5.4%, compounded monthly. She makes equal monthly repayments over 4 years.

(a) Find the monthly repayment amount.

(b) Find the total amount Priya pays over the 4 years.

(c) Find the total interest paid.

(d) After 2 years of payments, Priya wants to pay off the remaining balance in full. Using your GDC, find the outstanding balance at that point.

▶ Show Solution

WORKED EXAMPLE

Question 2 — Geometric Series in Context [~7 marks]

A biologist studying a bacterial culture observes that the population triples every 90 minutes. The initial population is 400 bacteria.

(a) Write a model for the population P after n intervals of 90 minutes.

(b) Find the population after 6 hours.

(c) Find how long it takes for the population to first exceed 1,000,000.

(d) The culture vessel has a capacity of 5,000,000 bacteria. Find the total number of bacteria across the first 8 intervals and comment on whether the vessel is sufficient.

▶ Show Solution

WORKED EXAMPLE

Question 3 — Logarithms in Exponential Growth Context [~6 marks]

The value V (in dollars) of a vintage guitar is modelled by $V(t) = 1200 \times e^{0.065t}$, where t is the number of years since it was purchased.

(a) Find the initial value of the guitar.

(b) Find the value after 10 years, correct to the nearest dollar.

(c) Find the number of years it takes for the value to double.

(d) The owner wants to sell when the guitar is worth at least $\$5000$. Find the minimum whole number of years they must wait.

▶ Show Solution