Geometry and Trigonometry

IB SL Study Guide

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IB Math AI SL — Geometry and Trigonometry

Complete Study Guide

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Topic 3 of the IB Math AI SL syllabus — Paper 1 and Paper 2

IB TIP

Practical geometry: This topic focuses on applying geometry to real scenarios: buildings, fields, navigation, urban planning. Always draw a clear diagram and label all known values before starting calculations. The IB awards marks for diagrams.

MEMORISE THIS

Essential formulas (all in the formula booklet)

Shape/Rule	Formula
Area of triangle	$A = \frac{1}{2}ab\sin C$
Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$
Volume of cylinder	$V = \pi r^2 h$
Volume of cone	$V = \frac{1}{3}\pi r^2 h$
Volume of sphere	$V = \frac{4}{3}\pi r^3$
Surface area of sphere	$A = 4\pi r^2$

Section 1: Mensuration

Mensuration is the measurement of lengths, areas, and volumes.

1.1 Area of 2D Shapes

Shape	Area
Rectangle	$A = lw$
Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Trapezium	$A = \frac{1}{2}(a+b)h$
Circle	$A = \pi r^2$
Sector	$A = \frac{\theta}{360}\pi r^2$ (degrees)

1.2 Surface Area and Volume of 3D Solids

Solid	Surface area	Volume
Rectangular prism	$2(lw + lh + wh)$	lwh
Cylinder	$2\pi r^2 + 2\pi rh$	$\pi r^2 h$
Cone	$\pi r^2 + \pi rl$ (where l = slant height)	$\frac{1}{3}\pi r^2 h$
Sphere	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Pyramid	base area + lateral faces	$\frac{1}{3} \times$ base area $\times h$

WORKED EXAMPLE

Mensuration — composite solid

A water tank consists of a cylinder of radius 1.2 m and height 3 m, topped by a hemisphere. Find the total volume and the total outer surface area.

Volume:

$V_{\text{cylinder}} = \pi(1.2)^2(3) = 4.32\pi = 13.57 \text{ m}^3$

$V_{\text{hemisphere}} = \frac{1}{2} \times \frac{4}{3}\pi(1.2)^3 = \frac{2}{3}\pi(1.728) = 1.152\pi = 3.619 \text{ m}^3$

$V_{\text{total}} = 13.57 + 3.619 = 17.2 \text{ m}^3$ (3 s.f.)

Surface area (outer only, no base):

$A_{\text{cylinder side}} = 2\pi(1.2)(3) = 7.2\pi$

$A_{\text{hemisphere}} = 2\pi(1.2)^2 = 2.88\pi$

$A_{\text{base}} = \pi(1.2)^2 = 1.44\pi$

$A_{\text{total}} = 7.2\pi + 2.88\pi + 1.44\pi = 11.52\pi = 36.2 \text{ m}^2$ (3 s.f.)

EXAM ALERT

Units matter: Always include units in your final answer. Area is in square units (m^2, cm^2), volume in cubic units (m^3, cm^3, litres). Remember $1 \text{ m}^3 = 1000$ litres.

Section 2: Right-Angled Trigonometry

2.1 Trigonometric Ratios (SOH CAH TOA)

For a right-angled triangle with angle θ:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

2.2 Pythagoras' Theorem

$$c^2 = a^2 + b^2$$

where c is the hypotenuse.

WORKED EXAMPLE

Right-angled trigonometry — building height

From a point 30 m from the base of a building, the angle of elevation to the top is 52 degrees. Find the height of the building.

Let h be the height.

$\tan 52^\circ = \frac{h}{30}$ $h = 30\tan 52^\circ = 30 \times 1.2799 = 38.4\ \text{m}$

2.3 Angles of Elevation and Depression

- **Angle of elevation:** measured upward from horizontal
- **Angle of depression:** measured downward from horizontal

These are **alternate angles**, so the angle of depression from point A to B equals the angle of elevation from B to A.

Section 3: Non-Right-Angled Trigonometry

When a triangle does not have a right angle, use the sine rule, cosine rule, or area formula.

3.1 Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use when you know: **a side and its opposite angle**, plus one other piece of information.

⚠ EXAM ALERT

Ambiguous case: When using the sine rule to find an angle, $\sin\theta$ can give two possible angles (one acute, one obtuse). Check whether both are valid in the context. This comes up frequently in exam questions.

3.2 Cosine Rule

To find a side: $c^2 = a^2 + b^2 - 2ab\cos C$

To find an angle: $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

Use when you know: **two sides and the included angle** (SAS), or **all three sides** (SSS).

WORKED EXAMPLE

Non-right trigonometry — surveying

Two points A and B are on opposite sides of a lake. From a third point C, the distances are $CA = 420$ m and $CB = 350$ m, and the angle $ACB = 73°$. Find the distance AB.

Using the cosine rule:

$AB^2 = 420^2 + 350^2 - 2(420)(350)\cos 73°$ $AB^2 = 176400 + 122500 - 294000 \times 0.2924$ $AB^2 = 298900 - 85966 = 212934$ $AB = 461$ m (3 s.f.)

3.3 Area of a Triangle

When you know two sides and the included angle:

$A = \frac{1}{2}ab\sin C$

WORKED EXAMPLE

Area — triangular field

A triangular field has sides 85 m and 110 m with an included angle of 64 degrees. Find its area.

$A = \frac{1}{2}(85)(110)\sin 64° = 4675 \times 0.8988 = 4202\ \text{m}^2 = 4200\ \text{m}^2$ (3 s.f.)

Section 4: Bearings and Navigation

4.1 Bearings

A **bearing** is measured clockwise from north, written as a three-digit number (e.g., 045 degrees, 270 degrees).

Direction	Bearing
North	000 degrees
East	090 degrees
South	180 degrees
West	270 degrees

4.2 Solving Bearing Problems

Strategy:

1. Draw a clear diagram with north lines at every point
2. Label all known angles and distances
3. Find the angles inside triangles by using properties of parallel lines (north lines are parallel)
4. Apply sine rule or cosine rule

WORKED EXAMPLE

Bearings — two ships

Ship A is 15 km from a lighthouse on a bearing of 040 degrees. Ship B is 22 km from the lighthouse on a bearing of 115 degrees. Find the distance between the ships.

The angle at the lighthouse between the two bearings: $115^\circ - 40^\circ = 75^\circ$.

Using the cosine rule:

$AB^2 = 15^2 + 22^2 - 2(15)(22)\cos 75^\circ$ $AB^2 = 225 + 484 - 660 \times 0.2588 = 709 - 170.8 = 538.2$ $AB = 23.2 \text{ km}$ (3 s.f.)

Section 5: Voronoi Diagrams

Voronoi diagrams are **unique to Math AI** — they do not appear in Math AA. A Voronoi diagram divides a plane into regions based on proximity to a set of points (called **sites**).

5.1 Key Concepts

- Each **cell** (region) contains all points closest to one particular site
- The **edges** of the diagram are sections of the **perpendicular bisectors** of the line segments joining neighbouring sites
- A **vertex** is a point equidistant from three (or more) sites

5.2 Constructing a Voronoi Diagram

For two sites A and B:

1. Find the midpoint M of segment AB

2. Find the gradient of AB, then the negative reciprocal (perpendicular gradient)
3. The perpendicular bisector through M is the Voronoi edge

For three or more sites, repeat for each pair and find where the perpendicular bisectors intersect.

WORKED EXAMPLE

Voronoi diagram — finding the perpendicular bisector

Two hospitals are located at $A(2, 5)$ and $B(8, 1)$. Find the equation of the Voronoi edge between them.

Midpoint: $M = \left(\frac{2+8}{2}, \frac{5+1}{2}\right) = (5, 3)$

Gradient of AB: $m = \frac{1-5}{8-2} = \frac{-4}{6} = -\frac{2}{3}$

Perpendicular gradient: $m_{\perp} = \frac{3}{2}$

Voronoi edge: $y - 3 = \frac{3}{2}(x - 5)$, i.e., $y = \frac{3}{2}x - \frac{9}{2}$ or $y = 1.5x - 4.5$.

5.3 Applications

Application	What the sites represent
Emergency services	Hospitals, fire stations — assign areas to nearest station
Retail	Shops — identify catchment areas
Ecology	Animal territories — nearest watering hole
Toxic waste	Dump sites — find the point farthest from all sites (Voronoi vertex)

IB TIP

The "toxic waste dump" problem: A classic Math AI question asks you to find the location to place a facility as **far as possible** from existing sites. The answer is always at a **Voronoi vertex**, because that is the point equidistant from three nearest sites, maximizing the minimum distance to any site.

5.4 Adding a New Site

When a new site is added to a Voronoi diagram:

1. Determine which existing cell contains the new site
2. Draw perpendicular bisectors between the new site and each neighbouring site
3. The new edges trim the old cell to create the new cell

⚠ EXAM ALERT

Nearest-neighbour interpolation: The IB may give you a Voronoi diagram with data values at each site and ask you to estimate the value at a given point. The answer is always the value at the **nearest site** (i.e., the site whose cell contains the point).

Section 6: Practice Questions

Paper 1 Style (Short Answer)

▶ **Q1.** A cone has radius 6 cm and slant height 10 cm. Find its volume.

▶ **Q2.** From the top of a 45 m cliff, the angle of depression to a boat is 28 degrees. Find the distance of the boat from the base of the cliff.

▶ **Q3.** In triangle PQR, $PQ = 12$ cm, $QR = 9$ cm, and angle $PQR = 108$ degrees. Find the area of the triangle.

Paper 2 Style (Extended Response)

▶ **Q4.** Three towns are located at $A(1, 3)$, $B(7, 1)$, and $C(4, 8)$. (a) Draw the Voronoi diagram for these three towns. (b) A new hospital is to be built equidistant from all three towns. Find its coordinates. (c) Find the distance from the hospital to each town.

▶ **Q5.** A hiker walks 4.5 km on a bearing of 065 degrees, then 3.2 km on a bearing of 140 degrees. (a) Find the direct distance from the starting point to the finishing point. (b) Find the bearing from the starting point to the finishing point.

⚠ EXAM ALERT

Bearing problems: Always draw north lines at every point. The most common error is getting the interior angle of the triangle wrong. Check by making sure all angles in your triangle sum to 180 degrees.

May 2026 Prediction Questions

⚠ EXAM ALERT

These are NOT official IB questions. These are trend-based practice questions written to reflect the topic areas and question styles most likely to appear on the May 2026 IB Math AI SL Paper 2. Based on recent exam patterns (2022–2025), expect heavy weighting on: 3D geometry combining Pythagoras and trigonometry, the cosine rule with triangle area, and bearing or navigation multi-part problems.

WORKED EXAMPLE

Question 1 — 3D Problem with Pythagoras and Trigonometry [~8 marks]

A telecommunications mast stands vertically on horizontal ground. The mast is 42 m tall. Two support cables are attached to the top of the mast and anchored to the ground. Cable A is anchored at a point 18 m from the base of the mast. Cable B is anchored at a point 25 m from the base, in a different direction.

(a) Find the length of cable A.

(b) Find the angle that cable A makes with the ground, correct to one decimal place.

(c) The two anchor points and the base of the mast form a triangle. The angle at the base between the two cable directions is 67 degrees. Find the distance between the two anchor points.

(d) Find the area of the triangle formed by the two anchor points and the base of the mast.

▶ Show Solution

WORKED EXAMPLE

Question 2 — Cosine Rule, Non-Right Triangle, and Area [~7 marks]

Two surveyors at points A and B are 340 m apart. From A, a landmark C is at an angle of 58 degrees from the line AB. From B, the landmark C is at an angle of 47 degrees from the line AB (on the same side).

(a) Find the angle at C in triangle ABC.

(b) Using the sine rule, find the distances AC and BC.

(c) Find the area of triangle ABC.

(d) A fence is to be built from A to C and from B to C. Find the total length of fencing required.

▶ Show Solution

WORKED EXAMPLE

Question 3 — Bearing and Navigation Problem [~7 marks]

A cargo ship leaves port P and sails 85 km on a bearing of 038 degrees to reach point Q. It then changes course and sails 60 km on a bearing of 124 degrees to reach point R.

(a) Draw a diagram showing P, Q, and R with north lines and all given information.

(b) Find the angle PQR.

(c) Find the direct distance from P to R.

(d) Find the bearing from P to R, correct to the nearest degree.

▶ Show Solution