Functions

IB SL Study Guide

Contents

IB Math AI SL — Functions

Complete Study Guide

Topics Covered

1. Function Concepts — domain, range, notation, graphs
2. Linear Models — gradient, equation of a line, direct variation
3. Quadratic Models — vertex form, optimization, projectile motion
4. Exponential Models — growth, decay, fitting data
5. Logarithmic Models — the inverse of exponential, pH scale, decibels
6. Sinusoidal Models — periodic phenomena, tides, temperatures
7. Modelling and Fitting — choosing the best model for data
8. Practice Questions and Exam Alerts

Topic 2 of the IB Math AI SL syllabus — Paper 1 and Paper 2

IB TIP

Modelling mindset: In Math AI, functions are introduced through real contexts, not abstract algebra. Every exam question gives you data or a scenario first, and expects you to identify or work with the appropriate model. Master the **shape** of each function type so you can recognize which model fits.

MEMORISE THIS

Model recognition at a glance

Pattern in data	Model type	Equation form
Constant rate of change	Linear	$y = mx + c$
One turning point, symmetric	Quadratic	$y = a(x - h)^2 + k$
Rapid increase/decrease, never zero	Exponential	$y = ka^x + c$
Inverse of exponential	Logarithmic	$y = a \ln x + c$
Repeating pattern	Sinusoidal	$y = a \sin(b(x - c)) + d$

Section 1: Function Concepts

A **function** is a relation where each input has exactly one output. In Math AI, you will usually work with functions as models of real situations.

1.1 Domain and Range

- **Domain:** the set of valid input values (x-values)
- **Range:** the set of possible output values (y-values)

In context, domain and range are restricted by the real-world situation. For example, if t represents time, typically $t \geq 0$.

1.2 Function Notation

$f(x) = 2x + 3$ means "the function f takes input x and outputs $2x + 3$."

$f(5) = 2(5) + 3 = 13$ means "when the input is 5, the output is 13."

1.3 Reading Graphs in Context

On the IB exam, you will be asked to interpret features of graphs:

Feature	What it means in context
y-intercept	Starting value (at $x = 0$)
x-intercept	When the quantity equals zero
Gradient (slope)	Rate of change
Maximum/minimum	Highest/lowest value of the quantity
Asymptote	A value the quantity approaches but never reaches

Section 2: Linear Models

A linear function has the form $y = mx + c$ where m is the **gradient** (rate of change) and c is the **y-intercept** (initial value).

2.1 Finding the Equation from Two Points

Given points (x_1, y_1) and (x_2, y_2):

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Then use $y - y_1 = m(x - x_1)$.

2.2 Direct and Inverse Variation

Direct variation: $y = kx$ (the graph passes through the origin). If you double x, you double y.

WORKED EXAMPLE

Linear model — taxi fare

A taxi company charges a fixed fee of 3.50 plus 1.80 per kilometre (dollars). Write a model for the fare F in terms of distance d, and find the fare for a 12 km trip.

$F(d) = 1.80d + 3.50$

$F(12) = 1.80(12) + 3.50 = 21.60 + 3.50 = 25.10$ (dollars)

The gradient $m = 1.80$ represents the **cost per km**. The intercept $c = 3.50$ is the **flag fall** (base charge).

2.3 Piecewise Linear Models

Many real situations use different linear rules in different intervals.

WORKED EXAMPLE

Piecewise function — phone plan

A phone plan charges 0.10 per MB for the first 500 MB and 0.05 per MB after that (dollars). Write a piecewise model for cost C *in terms of data usage* d *MB.*

$$C(d) = \begin{cases} 0.10d & \text{if } 0 \le d \le 500 \\ 50 + 0.05(d - 500) & \text{if } d > 500 \end{cases}$$

At $d = 500$: $C = 50$. At $d = 800$: $C = 50 + 0.05(300) = 65$.

Section 3: Quadratic Models

A quadratic function has the form $y = ax^2 + bx + c$. The graph is a **parabola**. If $a > 0$, it opens upward (minimum). If $a < 0$, it opens downward (maximum).

3.1 Vertex Form

$$y = a(x - h)^2 + k$$

The vertex is at (h, k). This form is most useful for optimization problems.

3.2 The Axis of Symmetry

For $y = ax^2 + bx + c$, the axis of symmetry is:

$$x = -\frac{b}{2a}$$

3.3 The Discriminant

$$\Delta = b^2 - 4ac$$

$\Delta > 0$	**Two distinct real roots (x-intercepts)**
$\Delta = 0$	One repeated root (vertex touches the x-axis)
$\Delta < 0$	No real roots (parabola does not cross the x-axis)

✎ **WORKED EXAMPLE**

Quadratic model — projectile motion

A ball is thrown upward from a platform 1.5 m above the ground. Its height h metres after t seconds is modelled by $h(t) = -4.9t^2 + 12t + 1.5$.

(a) Find the maximum height.

Using the axis of symmetry: $t = -\frac{12}{2(-4.9)} = \frac{12}{9.8} = 1.224$ s.

$h(1.224) = -4.9(1.224)^2 + 12(1.224) + 1.5 = -7.34 + 14.69 + 1.5 = 8.85$ m.

(b) Find when the ball hits the ground ($h = 0$).

Using the GDC to solve $-4.9t^2 + 12t + 1.5 = 0$: $t = -0.120$ or $t = 2.57$.

Since $t \geq 0$, the ball hits the ground at $t = 2.57$ seconds.

⚠ **EXAM ALERT**

Show your GDC setup: When the exam says "using technology" or does not say "without technology," you can use your GDC. However, always write the equation you are solving. For example, write "$-4.9t^2 + 12t + 1.5 = 0$, using GDC $t = 2.57$."

Section 4: Exponential Models

An exponential function has the form $y = ka^x + c$ where:

- k is the initial multiplier
- a is the base (growth if $a > 1$, decay if $0 < a < 1$)
- c is the horizontal asymptote

4.1 Growth and Decay

Exponential growth: population, compound interest, viral spread.

$$P(t) = P_0 \cdot a^t, \quad a > 1$$

Exponential decay: radioactive decay, cooling, depreciation.

$$Q(t) = Q_0 \cdot a^t, \quad 0 < a < 1$$

4.2 The Natural Exponential

The function $y = e^x$ where $e \approx 2.718$ is the natural exponential. In many models:

$$P(t) = P_0 e^{rt}$$

where $r > 0$ gives growth and $r < 0$ gives decay.

WORKED EXAMPLE

Exponential model — bacterial growth

A colony of bacteria has 200 cells at time $t = 0$ and doubles every 3 hours. Write a model for the number of bacteria N after t hours, and find when there are 10,000 bacteria.

$N(t) = 200 \times 2^{t/3}$

For $N = 10000$:

$200 \times 2^{t/3} = 10000$ $2^{t/3} = 50$ $\frac{t}{3} = \log_2 50 = \frac{\ln 50}{\ln 2} = 5.644$ $t = 16.9$ hours

4.3 Identifying Exponential Data

If the **ratio** between consecutive y-values is constant, the data is exponential.

x	0	1	2	3	4
y	100	120	144	172.8	207.4
Ratio	—	1.2	1.2	1.2	1.2

Constant ratio $\implies$ exponential model with $a = 1.2$.

Section 5: Logarithmic Models

The logarithmic function $y = \log_a x$ is the **inverse** of $y = a^x$.

5.1 Key Properties

- $\log_a 1 = 0$ for all bases a
- $\log_a a = 1$
- $\log_a(xy) = \log_a x + \log_a y$
- $\log_a(x^n) = n \log_a x$

5.2 Real-World Logarithmic Scales

Many real-world measurements use logarithmic scales because the quantities span many orders of magnitude:

Scale	Formula	Context
Richter scale	$M = \log_{10}\left(\frac{A}{A_0}\right)$	Earthquake magnitude
pH scale	$\text{pH} = -\log_{10}[\text{H}^+]$	Acidity
Decibels	$L = 10\log_{10}\left(\frac{I}{I_0}\right)$	Sound intensity

WORKED EXAMPLE

Logarithmic model — pH

The concentration of hydrogen ions in a solution is $[\mathrm{H}^+] = 3.2 \times 10^{-4}$ *mol/L. Find the pH.*

$$\mathrm{pH} = -\log_{10}(3.2 \times 10^{-4}) = -(\log_{10} 3.2 + \log_{10} 10^{-4}) = -(0.505 - 4) = 3.49$$

Section 6: Sinusoidal Models

Sinusoidal functions model **periodic phenomena** — quantities that repeat in a regular cycle. Examples include tides, temperatures, hours of daylight, and Ferris wheel height.

6.1 The General Sinusoidal Function

$$y = a\sin(b(x - c)) + d$$

Parameter	Meaning
a	Amplitude (half the distance from max to min)
b	Related to period: period $= \frac{2\pi}{b}$ or $\frac{360°}{b}$
c	Horizontal shift (phase shift)
d	Vertical shift (midline)

MEMORISE THIS

Quick parameter extraction from context

Given max value M and min value m:

$a = \frac{M-m}{2}, \qquad d = \frac{M+m}{2}$

If the period is T: $b = \frac{2\pi}{T}$ (radians) or $b = \frac{360}{T}$ (degrees).

WORKED EXAMPLE

Sinusoidal model — tidal height

The depth of water in a harbour varies between 2.1 m and 8.3 m. The time between successive high tides is 12.4 hours. High tide occurs at 03:00.

Write a model for the depth D at time t hours after midnight.

$$a = \frac{8.3 - 2.1}{2} = 3.1$$

$$d = \frac{8.3 + 2.1}{2} = 5.2$$

$$b = \frac{2\pi}{12.4} = 0.5067$$

High tide is at $t = 3$. Since sin reaches its maximum at $\frac{\pi}{2}$, we need $b(t - c) = \frac{\pi}{2}$ when $t = 3$.

Using cosine instead (max at $t = c$): $D(t) = 3.1\cos(0.507(t - 3)) + 5.2$

Check: $D(3) = 3.1\cos(0) + 5.2 = 3.1 + 5.2 = 8.3$ m. Correct (high tide).

IB TIP

Sine vs cosine: Use cosine when you know when the maximum occurs (since $\cos(0) = 1$). Use sine when you know when the midline crossing occurs (since $\sin(0) = 0$). Either is acceptable on the exam.

Section 7: Modelling and Choosing the Best Fit

A key Math AI skill is **choosing** which model best fits a dataset. Your GDC can fit regression models.

7.1 Steps for Modelling

1. **Plot the data** (scatter plot on GDC)
2. **Identify the shape** — linear, curved, periodic, etc.
3. **Run regression** on GDC for the candidate model type
4. **Check the fit** using R^2 (coefficient of determination)
5. **Interpret** the model parameters in context
6. **Evaluate limitations** — does the model make sense for large/small values?

7.2 Coefficient of Determination (R^2)

R^2 measures how well the model fits the data:

- $R^2 = 1$: perfect fit
- $R^2 \geq 0.9$: strong fit

- $R^2 \geq 0.7$: moderate fit
- $R^2 < 0.7$: weak fit

⚠ **EXAM ALERT**

Extrapolation warning: Models are only reliable within the range of the data used to create them. Predicting far beyond the data range (**extrapolation**) is unreliable. The IB often asks you to comment on the reliability of predictions — always mention whether you are interpolating (within range) or extrapolating (outside range).

✎ **WORKED EXAMPLE**

Choosing a model

The table shows the number of downloads D (thousands) of a new app over 8 weeks.

Week t	1	2	3	4	5	6	7	8
D	1.2	2.5	5.1	10.3	20.8	41.5	83.2	167

Check the ratios: $\frac{2.5}{1.2} = 2.08$, $\frac{5.1}{2.5} = 2.04$, $\frac{10.3}{5.1} = 2.02 \ldots$

The ratios are approximately constant at 2, suggesting an **exponential model**.

Using GDC exponential regression: $D = 0.598 \times 2.01^t$, $R^2 = 0.9999$.

This is an excellent fit. The downloads approximately double each week.

Limitation: This model predicts $D(20) = 0.598 \times 2.01^{20} = 660\,000$. In reality, growth would slow as the market saturates. The model is only reliable for the near term.

Section 8: Practice Questions

Paper 1 Style (Short Answer)

▶ **Q1.** A linear model for the cost of electricity is $C = 0.18u + 12.50$ where u is the number of units used. Interpret the 0.18 and the 12.50 in context.

▶ **Q2.** The temperature in a city varies sinusoidally. The maximum temperature is 34 degrees C in July and the minimum is 8 degrees C in January. Find the amplitude and the midline.

▶ **Q3.** A car worth 32,000 depreciates exponentially to 18,000 after 4 years. Find the annual depreciation rate.

Paper 2 Style (Extended Response)

▶ **Q4.** The depth of water D metres in a harbour at time t hours after midnight is modelled by $D(t) = 4.2\sin(0.507(t-3)) + 6.8$. (a) Find the maximum and minimum depth. (b) Find the period. (c) A boat needs at least 4 m of water. Find the times between midnight and noon when the boat cannot enter the harbour.

▶ **Q5.** The following data shows the number of users N (millions) of a social media platform. (a) Plot the data and suggest a suitable model. (b) Use your GDC to find the equation. (c) Predict the number of users in year 8. (d) Comment on the reliability of your prediction.

⚠ **EXAM ALERT**

Every extended-response modelling question on Paper 2 will ask you to comment on limitations or reliability. Marks are available for stating whether a prediction is interpolation or extrapolation, whether the model is appropriate for the long term, and what real-world factors might make the model break down.

May 2026 Prediction Questions

⚠ **EXAM ALERT**

These are NOT official IB questions. These are trend-based practice questions written to reflect the topic areas and question styles most likely to appear on the May 2026 IB Math AI SL Paper 2. Based on recent exam patterns (2022–2025), expect heavy weighting on: linear and exponential model fitting, domain and range in context, graph transformations, and interpreting points of intersection in real-world scenarios.

✎ **WORKED EXAMPLE**

Question 1 — Exponential Model Fit and Interpretation [~8 marks]

A wildlife reserve records the population of a deer species over several years.

Year (t)	0	1	2	3	4	5
Population (P)	320	374	438	512	599	701

(a) Show that an exponential model of the form $P = a \times b^t$ is appropriate for this data.

(b) Use your GDC to find the values of a and b, correct to 3 significant figures.

(c) Interpret the value of b in context.

(d) Use your model to predict the population in year 8.

(e) Comment on the reliability of your prediction in part (d).

▶ Show Solution

✎ **WORKED EXAMPLE**

Question 2 — Quadratic Model Optimization [~7 marks]

A football is kicked from ground level. Its height h metres after travelling a horizontal distance x metres is modelled by:

$$h(x) = -0.04x^2 + 1.2x$$

(a) Find the maximum height the ball reaches and the horizontal distance at which this occurs.

(b) Find the horizontal distance when the ball lands back on the ground.

(c) A goalkeeper is standing 28 m from where the ball was kicked. The crossbar is 2.44 m high. Determine whether the ball passes over the crossbar.

(d) State the domain of $h(x)$ in this context, giving a reason.

▶ Show Solution

✎ **WORKED EXAMPLE**

Question 3 — Piecewise Function Application [~6 marks]

An electricity provider charges customers based on daily usage in kilowatt-hours (kWh). The daily charge C dollars is modelled by:

$$C(u) = \begin{cases} 0.14u + 0.85 & \text{if } 0 \leq u \leq 20 \\ 0.22(u - 20) + 3.65 & \text{if } u > 20 \end{cases}$$

(a) Find the daily charge for a customer who uses 15 kWh.

(b) Find the daily charge for a customer who uses 35 kWh.

(c) A customer received a bill of $7.09 for one day. Find their usage.

(d) Explain what the value 0.85 represents in the model.

▶ Show Solution